Table 1 Fundamental frequency of a tapered beam and a plate tapered in one direction

	Beam		
	Fundamental frequency $\omega(m\ell/EI_0)^{1/2}$		
Taper ratio, α	Perturbation	Ref. 2	% Difference
.5	7.11	7.12	0.2
.67	5.97	6.06	1.5
.75	5.36	5.34	0.3
.90	3.99	3.88	2.8

	Plate		
	Fundamental frequency $\omega^2(\rho h_0 a^4/D_0)$		
Taper ratio, α	Perturbation	Ref. 3	% Difference
.2	748.2	748.2	0.00
.4	649.4	650.1	0.11
.6	556.6	557.2	0.10
.8	469.7	470.5	0.17

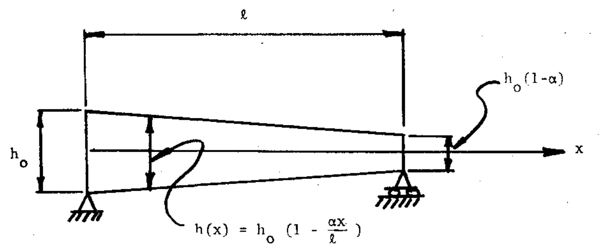

Fig. 1 Linearly tapered beam or cross section of a plate linearly tapered in the x direction.

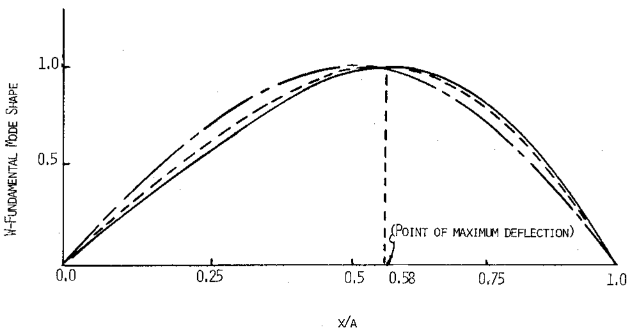

Fig. 2 Fundamental mode shape of a square plate linearly tapered in one direction (taper ratio $\alpha = 0.8$, $y = a/2$). Legend: (–) perturbation; (----) Ref. 3; (– – –) plate with uniform thickness.

III. Examples

A. Linearly Tapered Beam

The linearly tapered beam shown in Fig. 1 was selected as an example. The thickness variation can then be expressed as

$$\epsilon h_1(x) = -\alpha h_0 x/\ell;\ \epsilon = \alpha;\ h_1(x) = -h_0 x/\ell \tag{18}$$

The approximate fundamental frequency and mode shape are determined from Eqs. (9) and (10) and similar equations representing the second-order corrections. The change of fundamental frequency of the tapered beam is shown in Table 1 for four values of taper ratio and is compared with the exact frequency obtained from Ref. 2.

B. Linearly Tapered Plate

A simply supported square plate with linearly varying thickness in the x direction as shown in Fig. 1 was considered. The thickness of such a plate can be functionally represented as in Eq. (18). The approximate frequency for this particular plate with variable thickness was calculated from Eq. (16) and a similar equation representing a second-order correction. The frequency change for various values of taper ratio, α, is shown in Table 1 and compared with the frequency determined from Ref. 3. The fundamental mode shape as determined from Eq. (17) is shown in Fig. 2 and compared to the mode shape obtained from Ref. 3. The mode shape as determined from Eq. (17) is

$$w = [\sin(\pi x/a) + .1784\sin(2\pi x/a) + .004\sin(4\pi x/a) + .0005\sin(6\pi x/a)\sin(\pi y/a) \tag{19}$$

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Two-Dimensional Radiative Equilibrium: A Simple Nongray Problem

A. L. Crosbie*
University of Missouri-Rolla, Rolla, Mo.

A RECENT review[1] of the literature on two-dimensional radiative equilibrium reveals that all of the analyses use the gray approximation. The spectra of many substances, such as glass, carbon monoxide, and water vapor have windows or regions in which the absorption coefficient is zero or very small. The inability of the gray model to account for these windows is one of its major limitations. The present study of nongray radiative transfer in a two-dimensional

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*Professor, Thermal Radiative Transfer Group, Department of Mechanical and Aerospace Engineering. Member AIAA.

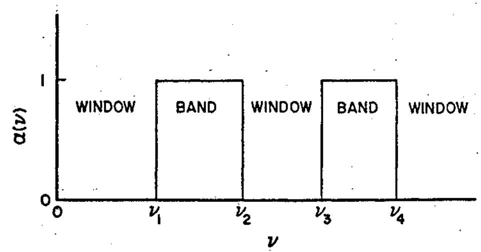

Fig. 1 Simplified rectangular model for the spectral absorption coefficient.

medium attempts to account for the windows with the simplified model shown in Fig. 1. The absorption coefficient is assumed to be only a function of frequency, i.e., $\kappa_\nu = \kappa\alpha(\nu)$, and the function $\alpha(\nu)$ is allowed only two values: zero or unity. The bands or continuum are approximated by rectangular boxes of equal height; however, the number, location, and width of the bands are unrestricted. This model has been applied successfully to one-dimensional planar[2] and spherical[3] problems, but has not been extended to two-dimensional geometries.

The coordinate system is illustrated in Fig. 2. The present investigation is based on the following assumptions: 1) two-dimensional transfer, 2) steady-state temperature and intensity, 3) absorbing-emitting but nonscattering medium, 4) local thermodynamic equilibrium, 5) no conduction or convection (radiative equilibrium), 6) refractive index of unity, and 7) absorption coefficient is independent of temperature, $\kappa_\nu = \kappa\alpha(\nu)$ with $\alpha(\nu)$ being unity or zero. For these assumptions, the transport equation is

$$\mathrm{d}I_\nu/\mathrm{d}\tau_s + \alpha(\nu)I_\nu = \sin\theta\,\sin\phi(\partial I_\nu/\partial\tau_y) + \cos\theta(\partial I_\nu/\partial\tau_z) + \alpha(\nu)I_\nu = \alpha(\nu)I_{b\nu}(T) \tag{1}$$

where $I_\nu(\tau_y, \tau_z, \theta, \phi)$ is the radiant intensity, $I_{b\nu}(T)$ is the Planck function, and T is temperature; θ and ϕ are the polar and azimuthal angles, respectively. The optical coordinates are defined as $\tau_s = \kappa s$, $\tau_y = \kappa y$, and $\tau_z = \kappa z$.

The intensity associated with the direction of increasing τ_z is denoted by I_ν^+ and that with decreasing τ_z by I_ν^-. The general boundary conditions for Eq. (1) are

$$I_\nu^+(\tau_y, 0, \theta, \phi) = I_{1\nu}(\tau_y, \theta, \phi)$$

$$I_\nu^-(\tau_y, \tau_0, \theta, \phi) = I_{2\nu}(\tau_y, \theta, \phi)$$

where $\tau_0 = \kappa L$ is the optical thickness. Application of integrating factor techniques to Eq. (1) yields the following expressions for I_ν^+ and I_ν^- in terms of the unknown Planck function

$$I_\nu^+(\tau_y, \tau_z, \theta, \phi) = I_{1\nu}(\tau_y^+, \theta, \phi)\,\exp[-\alpha(\nu)\tau_z\sec\theta] + \alpha(\nu)\int_0^{\tau_z} I_{b\nu}(\tau_y', \tau_z')\exp[-\alpha(\nu)(\tau_z - \tau_z')\sec\theta]\sec\theta\,\mathrm{d}\tau_z' \tag{2a}$$

where

$$\tau_y^+ = \tau_y - \tau_z\tan\theta\sin\phi \quad \text{and} \quad \tau_y' = \tau_y + (\tau_z' - \tau_z)\tan\theta\sin\phi$$

$$I_\nu^-(\tau_y, \tau_z, \theta, \phi) = I_{2\nu}(\tau_y^-, \theta, \phi)\,\exp[-\alpha(\nu)(\tau_0 - \tau_z)\sec\theta] + \alpha(\nu)\int_{\tau_z}^{\tau_0} I_{b\nu}(\tau_y', \tau_z')\exp[-\alpha(\nu)(\tau_z' - \tau_z)\sec\theta]\sec\theta\,\mathrm{d}\tau_z' \tag{2b}$$

where

$$\tau_y^- = \tau_y - (\tau_0 - \tau_z)\tan\theta\sin\phi \quad \text{and} \quad \tau_y' = \tau_y - (\tau_z' - \tau_z)\tan\theta\sin\phi \tag{2c}$$

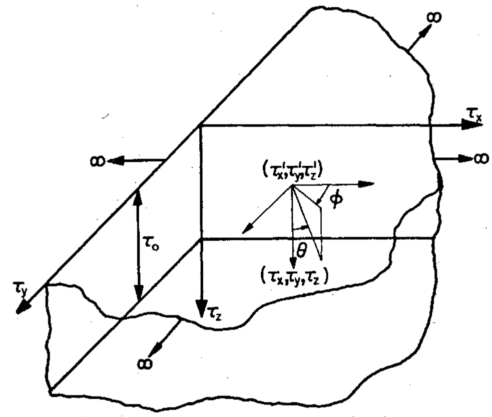

Fig. 2 Physical model and coordinate system.

The steady-state conservation of energy (radiative equilibrium) requires that

$$\nabla\cdot F = \int_0^\infty \kappa\int_{4\pi}\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_s}\mathrm{d}\omega\mathrm{d}\nu = \kappa\int_0^\infty \alpha(\nu)\int_{4\pi}[I_{b\nu} - I_\nu]\mathrm{d}\omega\mathrm{d}\nu = 0 \tag{3}$$

where F is the radiative flux and ω is solid angle. Substitution of Eq. (2) into Eq. (3) yields

$$4\pi\int_0^\infty \alpha(\nu)\,I_{b\nu}(\tau_y, \tau_z)\mathrm{d}\nu = \int_0^\infty\int_0^{2\pi}\int_0^{\pi/2} \alpha(\nu)I_{1\nu}(\tau_y^+, \theta, \phi)\exp[-\alpha(\nu)\tau_z\sec\theta]\,\sin\theta\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu$$
$$+ \int_0^\infty\int_0^{2\pi}\int_0^{\pi/2} \alpha(\nu)I_{2\nu}(\tau_y^-, \theta, \phi)\,\exp[-\alpha(\nu)\,(\tau_0 - \tau_z)\sec\theta]\,\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu$$
$$+ \int_0^\infty\int_0^{2\pi}\int_0^{\pi/2}\int_0^{\tau_0} \alpha^2(\nu)I_{b\nu}(\tau_y', \tau_z')\,\exp[-\alpha(\nu)\,|\tau_z - \tau_z'|\sec\theta]\,\tan\theta\,\mathrm{d}\tau_z'\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu \tag{4}$$

where

$$I_{b\nu}(\tau_y, \tau_z) = I_{b\nu}[T(\tau_y, \tau_z)] \quad \text{and} \quad \tau_y' = \tau_y - |\tau_z' - \tau_z|\tan\theta\sin\phi$$

In general, Eq. (4) is a nonlinear integral equation for the temperature distribution. However, when $\alpha(\nu)$ is restricted to values of zero or unity, the energy equation can be written as

$$4\,E(\tau_y, \tau_z) = \int_0^{2\pi}\int_0^{\pi/2} I_1(\tau_y^+, \theta, \phi)\,\exp[-\tau_z\sec\theta]\,\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi$$
$$+ \int_0^{2\pi}\int_0^{\pi/2} I_2(\tau_y^-, \theta, \phi)\,\exp[-(\tau_0 - \tau_z)\sec\theta]\,\sin\theta\mathrm{d}\theta\mathrm{d}\phi$$
$$+ \frac{1}{\pi}\int_0^{2\pi}\int_0^{\pi/2}\int_0^{\tau_0} E(\tau_y', \tau_z')\exp[-|\tau_z' - \tau_z|\sec\theta]\tan\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{5}$$

where

$$E(\tau_y, \tau_z) = \int_0^\infty \alpha(\nu)\,\pi I_{b\nu}(\tau_y, \tau_z)\mathrm{d}\nu$$

$$I_1(\tau_y, \theta, \phi) = \int_0^\infty \alpha(\nu)I_{1\nu}(\tau_y, \theta, \phi)\mathrm{d}\nu$$

and

$$I_2(\tau_y, \theta,\phi) = \int_0^\infty \alpha(\nu) I_{2\nu}(\tau_y, \theta,\phi)\mathrm{d}\nu \quad (6)$$

Equation (5) is possible because in the spectral region where $\alpha(\nu)=0$, all of the terms are zero. Thus, the energy equation is linear in E and is identical to the gray case.[4]

For collimated but nonuniform radiation incident on the upper surface

$$I_1(\tau_y, \theta,\phi) = I_0(\tau_y)\delta(\cos\theta - \cos\theta_0)\delta(\phi)$$

with

$$I_0(\tau_y) = \int_0^\infty \alpha(\nu) I_{0\nu}(\tau_y)\mathrm{d}\nu$$

and no radiation on the bottom $[I_2(\tau_y, \theta, \phi)=0]$, Eq. (5) can be solved as in the gray case,[4] i.e.,

$$E(\tau_y, \tau_z) = \int_{-\infty}^\infty g(\beta)\exp(i\beta\tau_y) J_\beta(\tau_y, \sigma_0; \tau_0)\mathrm{d}\beta \quad (7)$$

where $\sigma_0 = \sec\theta_0$ and

$$g(\beta) = \frac{1}{2\pi}\int_{-\infty}^\infty I_0(\tau_y)\exp(-i\beta\tau_y)\mathrm{d}\tau_y \quad (8)$$

and J_β satisfies the following one-dimensional integral equation

$$J_\beta(\tau_z, \sigma; \tau_0) = \exp(-\sigma\tau_z) + \frac{1}{2}\int_0^{\tau_0} J_\beta(\tau_z', \sigma; \tau_0)\mathcal{E}_1(|\tau_z - \tau_z'|, \beta)\mathrm{d}\tau_z' \quad (9)$$

with $\mathcal{E}_1(\tau,\beta)$ defined as

$$\mathcal{E}_1(\tau,\beta) = \int_1^\infty \exp[-\tau(t^2+\beta^2)^{1/2}]\mathrm{d}t/(t^2+\beta^2)^{1/2} \quad (10)$$

The integral equation, Eq. (9), has been studied elsewhere.[1,5] Likewise, when the medium is confined between two nonisothermal black walls $[\pi I_1(\tau_y, \theta, \phi) = E_1(\tau_y)$ and $\pi I_2(\tau_y, \theta, \phi) = E_2(\tau_y)]$, Eq. (5) can be solved, i.e.,

$$E(\tau_y, \tau_z) = \int_{-\infty}^\infty g_1(\beta)\exp(i\beta\tau_y)\phi_\beta(\tau_z; \tau_0)\mathrm{d}\beta + \int_{-\infty}^\infty g_2(\beta)\exp(i\beta\tau_y)\phi_\beta(\tau_0 - \tau_z; \tau_0)\mathrm{d}\beta \quad (11)$$

where

$$g_1(\beta) = \frac{1}{2\pi}\int_{-\infty}^\infty E_1(\tau_y)\exp(-i\beta\tau_y)\mathrm{d}\tau_y$$

and

$$g_2(\beta) = \frac{1}{2\pi}\int_{-\infty}^\infty E_2(\tau_y)\exp(-i\beta\tau_y)\mathrm{d}\tau_y \quad (12)$$

and $\phi_\beta(\tau_z;\tau_0)$ satisfies the following one-dimensional integral equation

$$\phi_\beta(\tau_z;\tau_0) = \frac{1}{2}\mathcal{E}_2(\tau_z, \beta) + \frac{1}{2}\int_0^{\tau_0}\phi_\beta(\tau_z'; \tau_0)\mathcal{E}_1(|\tau_z - \tau_z'|, \beta)\mathrm{d}\tau_z' \quad (13)$$

with

$$\mathcal{E}_2(\tau,\beta) = \int_1^\infty \exp[-\tau(t^2+\beta^2)^{1/2}]\mathrm{d}t/t^2 \quad (14)$$

The integral equation, Eq. (13), is identical to the one for the gray case.[1,4,5]

The z component of the radiative flux can be expressed as

$$q_z(\tau_y, \tau_z) = \int_0^\infty\int_0^{\pi/2}\int_0^{2\pi} I_{1\nu}(\tau_y^+, \theta, \phi)\exp[-\alpha(\nu)\tau_z\sec\theta]\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu$$
$$-\int_0^\infty\int_0^{2\pi}\int_0^{\pi/2} I_{2\nu}(\tau_y^-, \theta,\phi)\exp[-\alpha(\nu)(\tau_0 - \tau_z)\sec\theta]\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu$$
$$+\int_0^\infty\int_0^{2\pi}\int_0^{\pi/2}\int_0^{\tau_0}\alpha(\nu) I_{b\nu}(\tau_y', \tau_z')\mathrm{sign}(\tau_z - \tau_z')\exp[-\alpha(\nu)|\tau_z - \tau_z'|\sec\theta]\sin\theta\mathrm{d}\tau_z'\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu \quad (15)$$

Rearrangement of boundary radiation terms gives

$$q_z(\tau_y, \tau_z) = \int_0^\infty\int_0^{2\pi}\int_0^{\pi/2}[1-\alpha(\nu)] I_{1\nu}(\tau_y^+, \theta,\phi)\exp[-\alpha(\nu)\tau_z\sec\theta]\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu$$
$$-\int_0^\infty\int_0^{2\pi}\int_0^{\pi/2}[1-\alpha(\nu)] I_{2\nu}(\tau_y^-, \theta,\phi)\exp[-\alpha(\nu)(\tau_0 - \tau_z)\sec\theta]\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu$$
$$+\int_0^\infty\int_0^{2\pi}\int_0^{\pi/2}\alpha(\nu) I_{1\nu}(\tau_y^+, \theta,\phi)\exp[-\alpha(\nu)\tau_z\sec\theta]\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu$$
$$-\int_0^\infty\int_0^{2\pi}\int_0^{\pi/2}\alpha(\nu) I_{2\nu}(\tau_y^-, \theta,\phi)\exp[-\alpha(\nu)(\tau_0 - \tau_z)\sec\theta]\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu$$
$$+\int_0^\infty\int_0^{2\pi}\int_0^{\pi/2}\int_0^{\tau_0}\alpha(\nu) I_{b\nu}(\tau_y', \tau_z')\mathrm{sign}(\tau_z - \tau_z')\exp[-\alpha(\nu)|\tau_z - \tau_z'|\sec\theta]\sin\theta\,\mathrm{d}\tau_z'\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\nu \quad (16)$$

When $\alpha(\nu)$ is zero or unity the z-component of flux can be written as

$$q_z(\tau_y, \tau_z) = \int_0^{2\pi}\int_0^{\pi/2}[I_1^*(\tau_y^+,\theta,\phi) - I_2^*(\tau_y^-,\theta,\phi)]\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi$$
$$+\int_0^{2\pi}\int_0^{\pi/2} I_1(\tau_y^+,\theta,\phi)\exp(-\tau_z\sec\theta)\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi$$
$$-\int_0^{2\pi}\int_0^{\pi/2} I_2(\tau_y^-,\theta,\phi)\exp[-(\tau_0 - \tau_z)\sec\theta]\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi$$
$$+\int_0^{2\pi}\int_0^{\pi/2}\int_0^{\tau_0} E(\tau_y', \tau_z')\mathrm{sign}(\tau_z - \tau_z')\exp(-|\tau_z - \tau_z'|\sec\theta)\sin\theta\,\mathrm{d}\tau_z'\mathrm{d}\theta\mathrm{d}\phi \quad (17)$$

where

$$I_1^*(\tau_y, \theta,\phi) = \int_0^\infty [1-\alpha(\nu)]\, I_{1\nu}(\tau_y, \theta,\phi)\mathrm{d}\nu$$

and

$$I_2^*(\tau_y, \theta,\phi) = \int_0^\infty [1-\alpha(\nu)]\, I_{2\nu}(\tau_y, \theta,\phi)\mathrm{d}\nu \qquad (18)$$

The first term in Eq. (17) represents the net flux exchanged through the "windows." When the medium is gray, $\alpha(\nu)$ is unity and I_1^* and I_2^* are zero. The remaining terms are identical to those of the gray case.[4]

For the collimated boundary condition, the radiative flux becomes

$$q_z(\tau_y, \tau_z) = I_0^*(\tau_y)/\sigma_0 + \int_{-\infty}^{\infty} g(\beta)\, \exp(i\beta\tau_y)\, Q_\beta(\tau_z, \sigma_0; \tau_0)\mathrm{d}\beta \qquad (19)$$

where

$$I_0^*(\tau_y) = \int_0^\infty [1-\alpha(\nu)] I_{0\nu}(\tau_y)\mathrm{d}\nu$$

and

$$Q_\beta(\tau_z,\sigma;\tau_0) = \exp(-\sigma\tau_z)/\sigma + \frac{1}{2}\int_0^{\tau_0} J_\beta(\tau_z',\sigma;\tau_0)\, \mathrm{sign}(\tau_z-\tau_z')\, \mathcal{E}_2(|\tau_z-\tau_z'|, \beta)\mathrm{d}\tau_z' \qquad (20)$$

For the medium confined between black walls, the flux is

$$q_z(\tau_y,\tau_z) = q_0(\tau_y,\tau_z) + \int_{-\infty}^{\infty} g_1(\beta)\, \exp(i\beta\tau_y) F_\beta(\tau_z; \tau_0)\mathrm{d}\beta - \int_{-\infty}^{\infty} g_2(\beta)\, \exp(i\beta\tau_y) F_\beta(\tau_0-\tau_z; \tau_0)\mathrm{d}\beta \qquad (21)$$

where

$$F_\beta(\tau_z; \tau_0) = 2\mathcal{E}_3(\tau_z, \beta) + 2\int_0^{\tau_0} \phi_\beta(\tau_z'; \tau_0)\, \mathrm{sign}(\tau_z-\tau_z')\mathcal{E}_2(|\tau_z-\tau_z'|,\beta)\mathrm{d}\tau' z \qquad (22)$$

with

$$\mathcal{E}_3(\tau, \beta) = \tau\int_1^\infty \mathcal{E}_2(\tau x, \beta)\mathrm{d}x/x \qquad (23)$$

$q_0(\tau_y, \tau_z)$ represents the net flux exchanged through the "windows," i.e.,

$$q_0(\tau_y,\tau_z) = \int_0^{2\pi}\int_0^{\pi/2} [I_1^*(\tau_y^+) - I_2^*(\tau_y^-)]\cos\theta\sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi \qquad (24)$$

The energy equation has been reduced to a form identical with that of the gray analysis, whereas the expression for the z component of the radiative flux is the same as the gray case, except for a term accounting for the "windows." The influence of the "bands" occurs in functions E, I_1, and I_2, whereas the effect of the "windows" is accounted for by I_1^* and I_2^*.

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Solution Technique for Equilibrium Chemistry of Hydrogen-Oxygen Systems

David F. Dyer* and Glennon Maples†
Auburn University, Auburn, Ala.

Background

REFERENCES 1,2 describe an equilibrium chemistry model developed and used in conjunction with a computer program to predict the properties in a hydrogen-oxygen flame. The main features of the model are described.

For equilibrium reactions are assumed as follows

$$O + O \rightarrow O_2, \quad H + H \rightarrow H_2, \quad H + OH \rightarrow H_2O, \quad O + H \rightarrow OH \qquad (1\text{-}4)$$

The six species involved in these reactions are considered to be present with nitrogen which is inert. In developing the equations to predict the equilibrium concentration of the species, two quantities are defined, namely

$$X = m_{O_2} + m_O + \frac{W_O}{W_{H_2O}} m_{H_2O} + \frac{W_O}{W_{OH}} m_{OH} \qquad (5)$$

$$F = m_{H_2} + m_H + \frac{W_{H_2}}{W_{H_2O}} m_{H_2O} + \frac{W_H}{W_{OH}} m_{OH} \qquad (6)$$

where X is the total fraction of oxygen in any form, and F is the total fraction of hydrogen in any form. Since the molecular weight of the various oxygen species is approximately equal to that of nitrogen, it is assumed that the rate of diffusion of nitrogen is equal to that of the oxygen; and, therefore, nitrogen is present at any location in a fixed ratio to the fraction of oxygen compounds. This fraction, OFAC is assumed constant and equal to the fraction of oxygen in the air being used as the oxidizer. Thus,

$$X = (\mathrm{OFAC})(X + m_{N_2}) \qquad (7)$$

The total mass fraction must be unity which gives

$$X + m_{N_2} + F = 1 \qquad (8)$$

Substituting Eq. (7) into Eq. (8) and solving for X gives

$$X = (\mathrm{OFAC})(1\text{-}F) \qquad (9)$$

Therefore, if F is known, X can be determined by Eq. (9).

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*Associate Professor, Mechanical Engineering Department.

‡ Associate Professor, Mechanical Engineering Department. Member AIAA.